Functions Final Study Guide

**Chapter 4**

Section 4.1: Quadratic Functions and Models

* Know vertex form
  + f(x) = 2(x-3)2 + 1 Vertex = (3, 1)
* Vertex can be found at (-b/2a, c – b2/4a)

Section 4.2: Polynomial Functions and Roots

* Long division
* Remainders and factors
  + The remainder in polynomial division is 0 exactly when the divisor is a factor of the dividend.
* Remainder Theorem
  + If a polynomial f(x) is divided by x – c, then the remainder is the number f(c)
* Factor Theorem
  + The number c is a root of the polynomial f(x) exactly when x-c is a factor of f(x)
* Number of roots
  + A Polynomial of degree n has at most, n distinct roots
* The rational root test
  + Find R and S and then test the roots that the calculator suggests are roots by using division or remainder theorem

Section 4.2A: Synthetic Division

* Know how to do it

Section 4.3: Graphs of Polynomial Functions

* Know the graphs on page 307
* Knowing the shape of a polynomial will behave like its leading term when |x| is very large
  + In particular, when the polynomial function has odd degree, one end of its graph shoots upward and the other end downward
  + When the polynomial function has an even degree, both ends of its graph shoot upward or both ends shoot downward
* X-Intercepts
  + The graph of a polynomial function of a decree n meets the x-axis at most n times
* Multiplicity
  + Let c be a root of multiplicity k of a polynomial function f
    - If k is odd, the graph of f crosses the x-axis at c
    - If k is even the graph of f touches but does not cross the x-axis at c
* Local extrema
  + A polynomial function of degree n has at most n-1 local extrema. In other words, the total number of peaks and valleys on the graph is at most n-1
  + Section 4.3A: Optimization Applications
* Know how to create the equations that we will then optimize by finding the vertex

Section 4.5: Rational Functions

* Rational function is a function whose rule is the quotient of two polynomials
* The domain of the rational function f(x) = g(x) / h(x) is the set of all real numbers that are not roots of the denominator h(x)
* The Big-Little Principle
  1. If c is a number far from 0, then 1/c is a number close to 0. Conversely, if c is close to 0 then 1/c is far from 0.
  2. 1/big number = Little number
  3. 1/little number = big number
  4. This will help us see how the graphs behave when |x| is very large or very small
* For a linear rational function of the form f(x) = will have two asymptotes
  1. The vertical asymptote occurs at the root of the denominator
  2. The horizontal asymptote is the line y=a/c
* Properties of Rational Graphs
  1. Intercepts
     + The x-intercepts of the graph of the rational function f(x) = g(x)/h(x) occur at the number that are roots of the numerator g(x) but not the denominator h(x).
     + The y-intercept would occur at f(0) if there is one.
  2. Continuity
     + There will be no breaks in the graph of a rational function except for where the function is undefined
  3. Vertical Asymptote occurs at every number that is a root of the denominator h(x) but not of the numerator g(x)
  4. Horizontal asymptotes consider f(x) =
     + If n = k, then the line y=a/c is the horizontal asymptote
     + If n<k then the x-axis (the line y=0) is the horizontal asymptote
* Graphs of Rational Functions
  1. Analyze the function algebraically to determine its vertical asymptotes and intercepts
  2. Determine the horizontal asymptote of the graph when |x| is large by using the facts above.
  3. Use the preceding information to select an appropriate viewing window (or windows), to interpret the calculator’s version of the graph, and to sketch an accurate graph.

4.5A: Other Rational Functions

* Graphing f(x) = g(x)/h(x) when degree of g(x) is greater than degree of h(x)
  1. Analyze the function algebraically to determine its vertical asymptotes and intercepts
  2. Divide the numerator by the denominator. The quotient q(x) is the non-vertical asymptote of the graph, which describes the behavior of the graph when |x| is very large
  3. Use the preceding information to select an appropriate viewing window (or windows) to interpret the calculator’s version of the graph and to sketch an accurate graph

4.6: Complex Numbers

* The complex number system contains all real numbers
* Addition, subtraction, multiplication, and division of complex numbers obey the same rules of arithmetic that hold in the real number system, with **one exception**: the exponent laws hold for integer exponents, but not necessarily for fractional ones
* The complex number system contains a number *i* such that *i2 = 1*
* Every complex number can be written in standard form a + b*i*, where a and b are real numbers
* Two complex numbers a + bi and c + di are equal exactly when a = c and b = d
* The conjugate of the complex number a + bi is the number a – bi
* Square roots of negative numbers: the √-b = √bi (the square root sign should go over –b and bi)
* Every quadratic equation with real coefficients has solutions in the complex number system
* **Conjugate solutions:** if a + bi is a solution of a polynomial equation with real coefficients, then its conjugate a – bi is also a solution of this equation

**Chapter 5**

Section 5.1: Exponential Functions

1. Know what the graph of f(x) = ax for when a is both >0 and when 0<a<1
2. The function f(x) = Pakx is used for most exponential growth or decay questions
3. Inhibited population growth is depicted by an exponential equation that has either eto some coefficient or ato some coefficient on the bottom of a fraction (likely will not be on test but good to know just incase)

Problems for practice: Pages 377-379 #’s 1-13, 46, 48, 50

Section 5.2: Applications of Exponential Functions

1. Compound interest formula: A=P(1+r)t
   1. Know how to adjust that formula for compounded annually, quarterly, monthly, daily, etc.
2. Continuous compounding formula: A=Pert
   1. This will not be adjusted for different compound periods because it is continuously compounded
3. Radioactive decay is given by the formula M(x) = c(0.5x/h)
4. f(x) = Pax
   1. when a>1 it is exponential growth
   2. when 0<a<1 then exponential decay
   3. this function can have a coefficient “k” as described in section 5.1 above

Problems for practice: Pages 388-390 #’s 1-19, 20, 24, 26, 29, 30, 34, 45

Section 5.3: Common and Natural Logarithmic Functions

1. Definition of Common Logarithms: If c>0 then the common logarithm of c, denoted log c, is the solution of the equation 10x=c or in other words log c is the exponent to which 10 must be raised to produce c
2. Logarithmic and Exponential Equivalence: Let u and v be real numbers with v>0. Then log v = u exactly when 10u = v
3. Definition of Natural Logarithms: If c>0 then the natural logarithm of c, denoted ln c, is the solution of the equation ex=c or in other words ln c is the exponent to which e must be raised to produce c
4. Logarithmic and Exponential Equivalence: Let u and v be real numbers with v>0. Then ln v = u exactly when eu = v
5. **Properties of logarithms box on pg. 397**

Problems for practice: Pages 400-402 #’s 1-36

Section 5.4: Properties of Logarithms

1. Product law of logarithms: ln(vw)= ln v + ln w and log (vw) = log v + log w
2. Quotient law of logarithms: ln (v/w) = ln v – ln w and log (v/w) = log v – log w
3. Power law of logarithms: ln(vk) = k(ln v) and log (vk) = k(log v)

Problems for practice: Pages 408-409 #’s 1-24, 37

Section 5.4A: Logarithmic Functions to Other Bases

1. Definition of Logarithms to base b: if b and v are positive numbers, then the logarithm of v to base b, denoted logb v is the solution to the equation bx = v. In other words logb v = u exactly when bu = v.
2. Two boxes on page 413 that describe the properties of logarithms with base b and logarithm laws with base b
3. Change of base formula: for any positive numbers b and v, logb v = ln v / ln b

Problems for practice: Pages 417-418 #’s 1-36, 41-46, 51, 53, 57

Section 5.5: Algebraic Solutions of Exponential and Logarithmic Equations

1. If exponential equations have the same base we can set exponents equal to eachother
2. Read notes on how we solved these equations or refer to the examples on pages 422-425

Problems for practice: Pages 426-427 #’s 1-22, 25-42, 44, 48, 59

Section 5.7: Inverse Functions

1. The horizontal line test: if a function f is one-to-one then it has the following property. No horizontal line intersects the graph of f more than once.
2. Inverse functions definition: Let f be a one-to-one function. Then the **inverse function** of f is the function g whose rule is: g(y) = x exactly when f(x) = y. The domain of g is the range of f and the range of g is the domain of f.
   1. Remember if there is a limit on the range or domain of the original function than that is true for the opposite in g. If domain of f is all real numbers>0 than the range of g must be the same.
3. Finding inverse functions algebraically:
   1. Solve the equation f(x) = y for x.
   2. The solution is an expression in y, which is the rule of the inverse function g, that is, x = g(y)
   3. Rewrite the rule of x = g(y) as g(x) = y
4. **Round trip theorem: if you start with c and apply the original function and then the inverse you should have a final answer of c.** 
   1. g(f(x)) = x for every x in the domain of f
   2. f(g(x)) = x for every x in the domain of g
5. Inverse Function Graphs:
   1. if g is the inverse function of f, then the graph of g is the reflection of the graph of f over the line y=x
   2. If f is a one-to-one function and g is its inverse then (a,b) is on the graph of f exactly when (b,a) is on the graph of g
   3. f(a)=b then g(b)=a

Practice problems: Page 448 #’s 1-28, 35, 37, 39-44

**Chapter 9**

9.1: Trigonometric Functions of Acute Angles

* Sine – sin Ѳ =
* Cosine – cos Ѳ =
* Tangent – tan Ѳ =

9.2: Applications of Right Triangle Trigonometry

* Know how to draw a picture that is described in a word problem.